Using Action Research to Strengthen the Relationship between Literacy and Mathematics

"It was because of the wording nobody knew what to do"

- Year eleven math C student

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Abstract

This action research project was an investigation of the relationship between students' knowledge and use of mathematical vocabulary and their understanding of and ability to do math. Specifically, would an increased understanding and use of mathematical terminology help students to better understand the concepts and procedures to which those terms relate, and if so, would that increased understanding result in an increased ability to actually do math? A class dictionary of relevant mathematical terms was made and each student kept their own version in a small booklet. Written records of students' work were collected over a three week period and audio tape was used to record classroom conversations. The hope was that there would be an improvement in both the students' understanding of mathematical concepts and their ability to apply those concepts to solve problems. The students responded well to the math dictionary but many struggled to use the words to express their understanding. Severe time constraints placed significant limitations on the conclusions which could be drawn from this research. Nevertheless, the data suggested that while there was a distinct increase in students' understanding of the concepts associated with the terminology studied, more time was needed to demonstrate a clear link between an understanding of the terminology and an increased ability to do mathematics.

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Introduction

During my first lesson teaching a small year eleven math C class, a student asked a couple of questions regarding the meaning of some mathematical terms. Both “what’s a binomial?”[1] as well as “what’s a conjugate?”. The students had been exposed to both terms before since *binomials* are introduced in year nine, and the class had studied *surdic conjugates* earlier in the year. Perhaps, I thought, there is just not enough emphasis placed on the importance of learning the language of mathematics. The question proposed for this research was, would an increased understanding and use of mathematical terminology help students to better understand the concepts and procedures to which those terms relate, and if so, would that increased understanding result in an increased mathematical ability? An insightful comment from the authors of a modern mathematics textbook for year eleven offers some encouraging words to this approach and suggests the type of results which may be found:

> Mathematics…us[es] a highly refined form of language in which every word has an exact meaning…. As the structures and the logic of their explanation become more complicated, the language describing them in turn becomes more specialised, and requires systematic study for the meaning to be understood (Pender, Sadler, Shea, & Ward, 1999, p. 1).

As a result of this study and the application of the action research methodology, I hoped that as students internalized the vocabulary of mathematics they would make an associated increase in their understanding of and ability to do mathematics. This expectation is based on the idea that language is not only a vehicle for communication, but is the determining factor in raising consciousness. Neil Postman puts it succinctly when he says: “To speak new words in new ways is not a cosmetic activity. It is a way of becoming a new person” (1979, p. 151).

[1] Note: all students’ comments have been recorded in my action research journal which is included in the list of references under: (van Tol, 2008).

The time constraints for the research were very tight. Everything needed to be done in just three weeks due to a week of work experience undertaken by all year eleven students, as well as almost two weeks spent on the revision, writing, and handing back of exams. My intention was to investigate students' knowledge of and ability to use the mathematical terminology pertinent to the areas of study, attempt to improve their vocabulary through the use of that terminology, and then compare their comprehension of and ability to solve problems involving those terms before and after the intervention. [Note: A glossary of all mathematical terms appears at the end of this paper. All such terms are written in italics].

Literature Review

Mathematics Education

Mathematics has played an important role in the education system stretching back to that provided by the Greeks, including Plato and Pythagoras (Pottage, 1994). Throughout the Middle Ages, mathematics, or more specifically arithmetic and geometry, formed a key part of the liberal arts education program (Gullberg, 1997, p.870). The birth of science and the popularization of empiricism provided the foundation for the modern philosophical notion of the mind as *tabula rasa*, or, a blank slate, which had important implications for education across all subjects including mathematics. In arithmetic, Thorndike's emphasis on rote learning and drills (English & Halford, 1995, pp. 2-3) was the result; one had only to 'etch' the rules onto the slate of the mind. This view changed with the rise to prominence of constructivist theories of learning which posit that learners construct their own knowledge. In mathematics, constructivism became accepted as it grew out of Intuitionism whose main proponent was L.E.J. Brouwer (Hersh, 1994, p. 12). Although constructivism is a theory of learning rather than teaching, it has transformed mathematics education greatly through the use of manipulatives and class discussions (English & Halford, 1995, pp. 11-12).

The cultural dependence of mathematics and the possibility that different cultures can produce different mathematics has become quite an accepted view (Bishop, 1998; Brown, 1994). Perhaps the most powerful recent impact of modern western culture on mathematics education has come from the computer and graphing calculator. The computer has no doubt brought about a huge epistemological change in students' relationship to mathematics, though its ability to improve students' grades is far from certain (Wainer, Dwyer, Dutra, Covic, Magalhaes, Ferreira, Pimenta, & Claudio, 2008). Graphing calculators are now becoming commonplace in mathematics classrooms of all levels (Reznichenko, 2007, p. 6). Though advanced procedures may be undertaken with the use of such technology, the systematic requirement of students to use such technology expends time which could be used to develop other mathematical skills (Forster, 2006). To a large extent, the problem is that “teachers cannot exploit new technology in their daily practice if they are not well informed on its place and role in a didactical process” (Balacheff & Kaput, 1996, p. 486), while at the same time not having the authority to make the decision of such technology’s inclusion or exclusion.

Curriculum Policy

The Principles and Standards for School Mathematics (2000) published by the National Council for Teachers of Mathematics includes ‘Communication’ as one of their five process standards in which they state, “when students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language” (p. 4). The Queensland Mathematics C syllabus also makes numerous references to the importance of communication in mathematics and Section 7 is specifically called “Language education”. Therein it states that students should be able to “use the specialised vocabulary and terminology related to Mathematics C” (Queensland Studies Authority, 2008, p. 44). In addition, according to the syllabus, students are to be assessed in three categories, one of which is ‘Communication and Justification’.

The Language of Mathematics

The traditional notion of mathematics as an abstract and absolute subject has informed mathematics education for a very long time as evidenced by the transmission-based pedagogy which has been the norm in most classrooms until recently. There is now a growing body of research which suggests that a sociolinguistic approach to mathematics is not only possible, but desirable (see for example, Ernest, 1994; Schoenberger & Liming, 2001; Blessman & Myszczak, 2001). My own belief that the language of mathematics should be a major component of any lesson began when I used the term *asymptote* in a year eleven class to which one student retorted in an exasperated voice: “Asymptote! Isn’t there an easier word for it?”

Paul Ernest is a prominent academic promoting mathematics as a sociolinguistic subject. Ernest takes the view that "in deep and multiple ways…mathematics is at base conversational" (1994, p. 35). However, the difficulty in acquiring a proper mathematical vocabulary is due to math having "more concepts per word, per sentence, and per paragraph than any other area" (Schell, as cited in Monroe & Panchyshyn, 1995, p. 80). In addition, the way in which students read a math problem is not necessarily from left to right since attention needs to be diverted to diagrams, graphs, and tables. This type of attention is significantly different from that required to read normal text (Bosse & Faulconer, 2008, p. 9).

The importance in mathematics of actually understanding the concepts and procedures cannot be overstated. Without this understanding, any student who attempts mathematics will find high achievement virtually impossible. In fact, some students do not even realize that there *are* concepts behind the procedures that they use (Oaks, as cited in Porter & Masingila, 2000, p. 165). Students will definitely need to demonstrate this understanding in explaining reasons for their solutions. McMillan concurs when he states "an effective approach for fostering deep understanding is to ask students to justify and explain their answers" (2007, p. 211). Thus, beyond getting students simply to master the spelling of mathematical terms or memorize their definitions, students should be encouraged to use the terms to express their understanding. Though many teachers may feel that they are already short on time in class, the incorporation of writing need not take more than a few minutes, yet can still be engaging (Ryan, Rillero, Cleland, & Zambo, 1996). Perhaps most importantly from a teacher's perspective however, is the idea that as teachers we need to foster a cross-discipline enjoyment of learning ourselves in order to inculcate the same values in our students (Duncan, 1997, p. 9).

Numerous studies have tried using math journals and dictionaries to get students to start putting their understanding of mathematics into words (Clarke, Waywood, & Stephens, 1993; Schoenberger & Liming, 2001; Schwarz, 1999; Powell, 1997). Some studies employing this approach have shown that as students increase their use and understanding of mathematical language, their affect towards math increases as well (see for example, Blessman & Myszczak, 2001). Other studies using math journals produced positive results in relation to students' mathematical ability, as well as their affect (see for example, Schwarz, 1999).

Many studies at the elementary and junior level show a positive correlation between students' language and math ability (see for example, Ntenza, 2006; Goldsby & Cozza, 2002). At the senior level, Porter and Masingila (2000) have concluded that actually writing about mathematics concepts yields similar results to students simply discussing them, and that any exercise which involves students spending time reflecting on and communicating their ideas about mathematics will produce an increase in their ability to do math. At the college level, students who used a language-based method to study mathematics compared to a control group who did not, while only producing a small increase in outcomes, showed great enthusiasm for the technique (Lesnak, 1993).

A great deal of research has focused on the improvements made from taking a linguistic approach to mathematics as a large group (Kasparek, 1996; Jurdak & Abu Zein, 1998; Schoenberger & Liming, 2001). Fletcher and Santoli (2003), whose research participants were a small group of gifted students, conclude that an emphasis on mathematical language effected an increase in their students' comprehension. Their group appears fairly similar to my small class of year eleven math C students, most of whom seem quite bright. I thought it would be interesting to see if I could substantiate the findings of Fletcher and Santoli as I have not come across any other research done in a similar setting.

Methodology

Action Research

The reasons for using an action research approach for this study are varied as many of the characteristics of action research appeal to my values. This is important since "action research is value laden" and it "often begins by articulating your values and asking whether you are being true to them" (McNiff & Whitehead, 2006, p. 23). I believe one of the fundamental questions any good educator needs to pose to himself is, to what extent should I educate for conformity and to what extent should I educate for change? The answer to this question will vary from person to person not only in degree, but also with regards to the object of conservation or change. Given that I see numerous ills in modern life, one of the most significant of which I have expanded upon below, and also that inherent in both the name and methodology of action research is active change (McTaggart, 1997, p. 34), action research is a good technique for informing my teaching praxis.

One of the specific advantages of action research is its emphasis on flexibility which makes an associated demand for creativity (Dick, 1993, ¶ 25). This suits me well as I relish opportunities to find imaginative solutions to practical problems. This was the reason I studied engineering during my undergrad only to end up after graduating working at jobs which, in my case, required little more than copying drawings. While I have found teaching nurtures my affinity for creativity and adaptability far more than engineering, I have also found it more congenial in terms of its social nature. Even though while teaching the focus of action research is often oneself (Koshy, 2005, p. 10), this is not to be interpreted in a solipsistic fashion since, action research also emphasizes the dialectical relationship between the researcher and participants as they seek to create knowledge together (McNiff & Whitehead, 2006, pp. 26-27). In this manner, I have found teaching, and in particular using action research while teaching, to help me focus on "living in the direction of [my] values" (McNiff & Whitehead, 2005, p. 93).

Throughout action research is the recurring theme of 'social justice'. Given its importance, defining the term would be prudent. According to the Jesuits, social justice:

> focuses on social class inequities: putting the needs of the poor and vulnerable first, transforming the role of the economy to better serve people, emphasizing the right of all people to be treated with dignity and to engage in productive work with decent and fair wages, and to organize and join unions (Cuban & Anderson, 2007, p. 145)

Another definition, which is related but somewhat different, is given by the social justice educators, Adams, Bell, and Griffith. They define a socially just society

> as one in which the distribution of resources is according to need so that all members have their basic needs met. In addition, all members are physically and psychologically safe and secure, are able to develop their full capabilities and are capable of interacting democratically with others. All people also have a sense of their potential and actual power as well as a sense of social responsibility toward others and society as a whole (as cited in Cuban & Anderson, 2007, p. 146).

Finally, perhaps the first ever written definition of justice was given by Plato when he wrote: "when the workers, the military, and the guardians keep to their own work – the work they are naturally fitted for – *that* is justice and that makes the society just" (Plato, trans. 1966, p. 78, emphasis in original).

Each of these definitions are unique. The first contains a heavy focus on economics, the second an emphasis on individual wellbeing and democratic relationships, and the third stresses individuals fulfilling their natural roles. Although many action researchers tend toward the second definition with its emphasis on political and social relationships (see for example, McNiff & Whitehead, 2002, p. 90; McTaggart, 1997, pp. 36-37), there are others who take a somewhat more even-handed approach to balancing political and economic considerations (see for example, Noffke, 1995, p. 1 ff). John Dewey, who many regard as providing the intellectual vanguard to the action research movement (Burns, 1999, p. 26; Shubert & Lopez-Shubert, 1997, pp. 203, 210-214; Adelman, 1997, p. 84), supported the essence of Plato's theory of education, and in so doing, his notion of justice. Of course, Plato despised democracy, and not without providing good reasons (trans. 1966, pp. 154-155). Plato thought that education's role in a just society should be to help each individual discover his or her true nature, and thus to determine to which of the three classes they belonged in the society he envisioned. This basic purpose of education Dewey agreed with, but where he departed from Plato's reasoning was where he saw each individual as forming a class of his or her own (1966, pp. 88-90). Dewey was clearly in favour of democracy, but he understood that "politics is the shadow cast on society by big business" (Westbrook, as cited in Chomsky, chap. 9)[2], and for this reason I am most concerned with the first definition of social justice which emphasizes economics. I will elaborate on this more below.

Participants

The research project was conducted at a state school of approximately 500 students in a metropolitan area during my internship. The group was a year eleven math C class consisting of three girls named Desiree, Annie, and Meredith, and three boys named Marco, Steve, and Chad[3], all of whom were quite bright. I had two supervising teachers throughout the internship; one for math and another for science. My math supervisor, named Jessica, was the acting head of department and my science supervisor's name was Dave. I am a second year master of teaching student who enjoys learning and likes to read. My nominal teaching areas are physics and math, but I have also taught English as a Second Language (ESL) for about two and a half years and thus, this research project is a great way for me to combine my interests.

The procedure described below needed to be adjusted slightly as the research proceeded due to unforeseen obstacles. Such adjustments are typical of action research (Bartlett, 2003). The data collection tools I used in each step of the research are explained respectively below.

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[2] This book has an annotated bibliography which is available online only and which is organized by chapter only. Both the book and the online reference have been given in the list of references below.
[3] Names of all participants in this paper are pseudonyms.

Reconnaissance Stage

I began my research with a reconnaissance stage by recording my observations of the class in my research journal while my supervisor Jessica was taking most of the lessons. During my first lesson teaching the class, I used the term ‘binomial’ to which Marco responded “what’s a binomial?”. A little later during the same class Marco asked what a *conjugate* was. From these early stages during my internship, I started thinking perhaps a lack of mathematical terminology was hindering the students’ understanding of mathematics itself and I endeavoured to continue monitoring for any other evidence which would support this view.

Later, in order to explicitly check the students’ mathematical vocabulary, I gave them a little vocabulary quiz. Being an ESL teacher, and from having studied both Japanese and Spanish on my own, I knew there could be a significant difference between a person’s active and passive vocabulary. For this reason, I designed the quiz in two parts. The first part checked the students’ active vocabulary by asking them to provide definitions for ten different mathematical terms which they had covered earlier in the year, and most of which were necessary for the unit on vectors which we were about to commence. The second part assessed the students’ passive vocabulary by simply asking them to match twelve mathematical terms with a list of definitions provided. There were more definitions than terms in order to keep the students from using process of elimination to match terms with some of the more difficult definitions. Also, since a few of the terms were used in both parts, the active vocabulary was completed first so as not to give away any of the definitions. In addition to these two small quizzes, I gave the students a brief survey regarding the means by which they usually build their vocabulary.

Having analyzed the data which substantiated my initial thoughts that perhaps it was a lack of mathematical vocabulary which was impeding the students from excelling in the course, I started planning for my first action cycle.

Cycle 1

Based on some of the techniques trialed in the literature, my *plan* was to begin keeping a class dictionary of any new or relevant mathematical terms which the class encountered. When I *implemented* this plan, for each entry I listed the term, its definition which the students and I created together, as well as an example of the symbolism related to the term. Each of these three components were entered in a different colour for ease of reference. The dictionary was kept on large pieces of chart paper which I borrowed from the art department so that I could hang each page around the room. I also encouraged the students to keep their own version of the dictionary at the back of their notebook so they could refer to it at home while doing their homework.

Based on my experience as a language teacher and student, I knew it would be crucial to give the students the opportunity to use the terminology, either in writing or speaking, to express their understanding of the concepts and procedures. As such, I had also *planned* to incorporate time during the lessons for this purpose. I recorded all my *observations* in my research journal. At the end of this cycle, I took a few minutes at the end of class to ask the students what they thought about what we had been doing so far. Their feedback was that they all liked the idea of keeping a math dictionary, but they could not (or perhaps would not) provide any criticisms of the technique, nor did they offer any suggestions as to how we might improve what we were doing. As mentioned above, the time constraints on this research project were very tight, so this cycle only lasted about one and a half weeks. *Reflecting* upon my observations and the feedback the students had given, though I agreed that the dictionary was working well, the students had had very little opportunity to use the language as I had planned, again, due to the time restrictions. In the four classes during the first cycle, I only managed to fit in one ten-minute activity where the students were using the language in pairs to investigate a problem. This lack of vocabulary usage affected the changes I made to my praxis for cycle two.

Cycle 2

Based on my reflections from cycle 1 that there would not be sufficient time in class to do anything but continue adding to the dictionary, I *planned* to give the students a few extra problems to solve for homework and ask them to include written descriptions of their understanding (or misunderstanding) in their solutions. When I *implemented* this, I created the problems myself and deliberately made them challenging ones so as to stretch the students' abilities. I told the students that I would provide written feedback on whatever they handed in to me, and in this way we could keep a conversation going between us using the target language. In addition, since I found that many of the details of the class interactions escaped me when I tried to write my *observations* during the class, I got permission to make audio recordings of my lessons from the internship coordinator at my school.

Another teacher had offered a few mini-notebooks in which my students could keep their dictionary. At first I thought they would not be interested in having a separate notebook to carry around, but when I offered it to them they all eagerly accepted the notebooks and went about transcribing their dictionary into it. Upon *reflection,* the dictionary seemed to still be working very well. However, the same could not be said for the students' opportunity to use the target language. Although we were no longer using our very limited class time to practice using the terms in the dictionary, the students were bogged down with homework from their other subjects and preparing for their one week of work experience. I worked vigorously to return promptly anything the students handed in to me with my written comments. I used the target language to show them where they had gone wrong and to make suggestions for what they might try instead. Although I had hoped for more progress I did get some positive results, and in light of the time limits, perhaps the outcomes were not too disappointing.

Validity

Regarding the validity of the data collected, validity can be defined as a measure of the accuracy of the inferences a teacher can draw from an assessment (McMillan, 2007, p. 64). Though validity is generally broken down into the three subcategories of content-, criterion-, and construct-validity, (McMillan, 2007, pp. 65-69; Popham, 1995, pp. 42-55) all of them are increased as the amount and variety of evidence grows. As McMillan states: "to enhance validity, you need to use multiple assessment methods for each learning target and look for consistency with these results" (2007, p. 111). The means by which I attempted to validate my data were by collecting data in both written and verbal form, and by gathering data from all of the students in the class, and then looking for regularities which I could use as evidence. I would have liked to accumulate more data, but given the time constraints this was not possible.

Results

Discerning any appreciable increase in students' actual ability from their grades is difficult, even in mathematics which tends to be more objectively assessed. Despite the recent effort to make assessment more valid and reliable, there are still numerous factors that may not be accounted for by simply looking at grades, including, changes in students' daily affect, test bias, and most significantly, calling upon problem solving skills under artificial circumstances. Nevertheless, Table 1 below shows the grades of the class before, during, and after the intervention. With the exception of a very slight decrease in Desiree's Knowledge and Procedures (KaPs) marks from the semester 1 to the mid-semester 2 exam, and Marco's Modelling and Problem solving (MaPs) marks, and Steve's Communication and Justification (CaJ) marks over the same two exams, all the grades remained unchanged or increased.

Name	End of Semester 1 Exam (before the intervention)			Semester 2 Assignment (during the intervention)			Mid-Semester 2 Exam (after the intervention)		
	KaPs	MaPs	CaJ	KaPs	MaPs	CaJ	KaPs	MaPs	CaJ
Annie	64%	B+	B	100%	B-	A	75%	B+	A
Chad	28%	D+	C	83%	B+	B	33%	C+	C
Desiree	56%	C	C	83%	C+	B	54%	B+	B
Marco	34%	C	C	90%	C	B	51%	C-	C
Meredith	41%	C	C	97%	C+	A	59%	B-	B
Steve	52%	D+	B/C	87%	C-	B	57%	C+	C

Table 1: Assessment results before, during and after the intervention

Although the semester 2 assignment which the students completed during the intervention shows a great increase in grades from the semester 1 exam, the circumstances under which it was completed were totally different than the exam and therefore disqualify it from indicating any improvements in the students' mathematical ability. For the same reason we should not be concerned that the students' grades appeared to fall from the assignment to the mid-semester 2 exam.

Hence, there appears to be an overall slight increase in the students' grades over the course of the intervention, and there is certainly no important decrease in any of the grades. However, the most alluring evidence that perhaps there is a connection between the language and symbolism of mathematics came from qualitative rather than quantitative data.

One observation which led me to think the students would benefit from some emphasis placed on mathematical terminology came from Marco when he raised one of the homework questions which asked to determine if the group {0, 1, 2, 3, 4} could form a *cyclic group* under the operation of *addition modulo 5*, and if so, what the *generators* were. This question was loaded with highly specialized vocabulary and I had to do a bit of research on my own before the class to ensure that I could answer it effectively. With a little probing I found that Marco thought *addition modulo 5*, meant add 5, which is not only incorrect, but misleading.

The results of the reconnaissance stage which encouraged me to carry out this research were strongly suggestive. In my experience as a foreign language teacher and student, people's passive vocabulary is often stronger than their active vocabulary and the results of the reconnaissance stage demonstrated this well. Even though I was quite lenient when considering the students' definitions for the terms in the first part of the vocabulary quiz, on average only about two out of ten were answered correctly with no student answering more than three out of ten correctly. Even though Desiree had mixed up the definitions of the *associative law* and the *commutative law*, I considered them correct anyway. I noted a similar confusion of terminology during one of the first lessons I taught when Annie was giving the solution to a problem and said to "multiply by the determinant…or whatever it is". I knew she meant *discriminant* instead of *determinant* even though the two words refer to entirely different concepts. Many of the definitions given for a *radian* on the quiz were akin to Josh's "a unit of measure of an angle" which, while not incorrect, misses the essence of how a radian is defined. However, when it came to the matching exercise which only examined passive vocabulary, the students did much better by answering about seven out of twelve correct on average. Furthermore, although nobody could give a precise definition of what a *radian* was, four out of the six students were able to identify the definition of a *radian* from the list provided. Finally, on the vocabularybuilding survey most of the students said that they try to retain new words they learn by using them in writing except for Annie and Chad who said they preferred using them in speech, and Desiree who said she used both.

Perhaps the single most insightful piece of evidence I found which suggested that there was a relationship between the language, understanding, and symbolism of mathematics came from Marco when he was working on one of the homework problems I created. I asked the students to solve it in the usual symbolic manner but also to describe their reasoning in words. Marco started out by drawing a diagram and then producing an equation to link the sides of what he thought was a right-angled triangle as shown below in Figure 1.

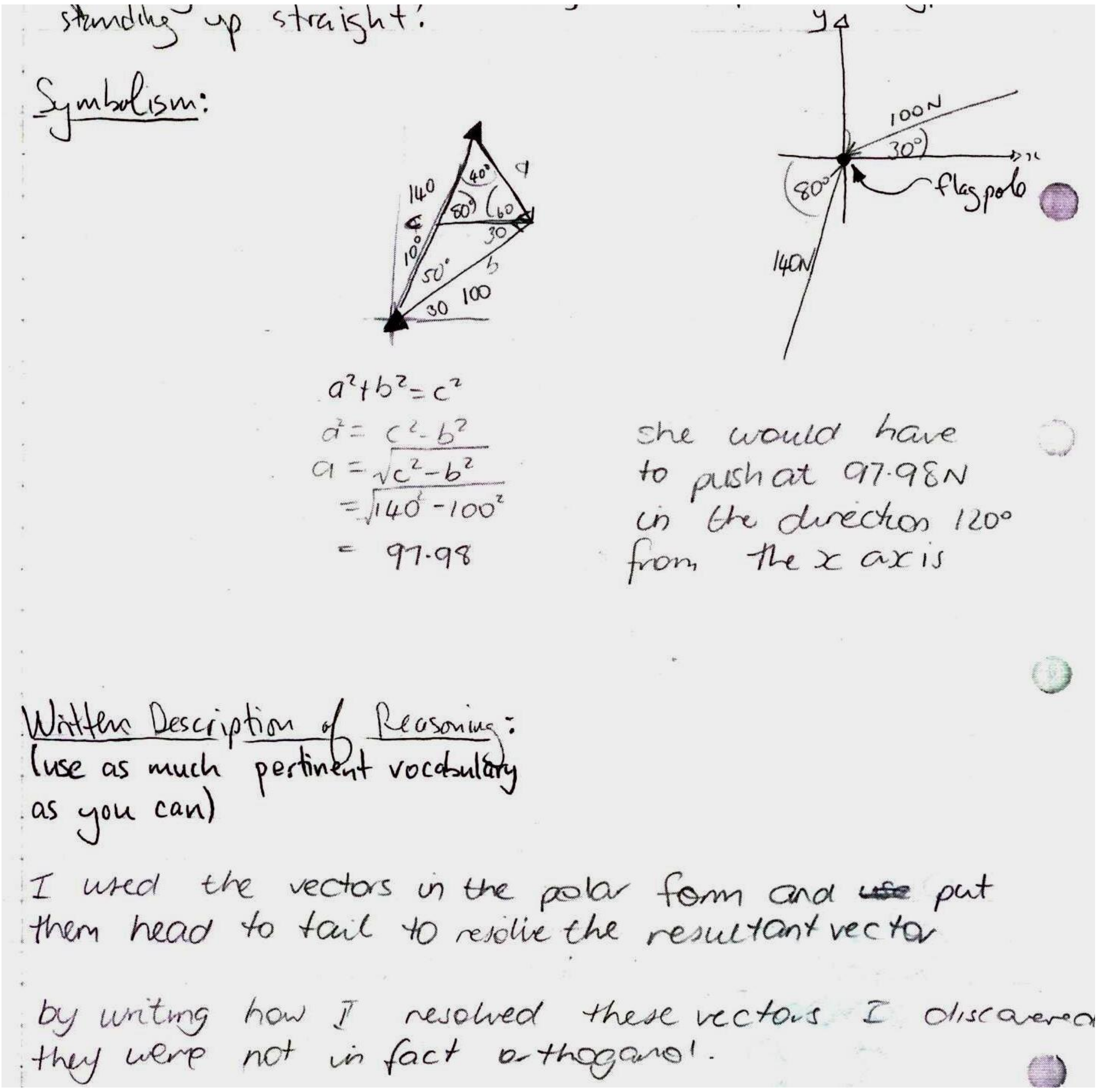
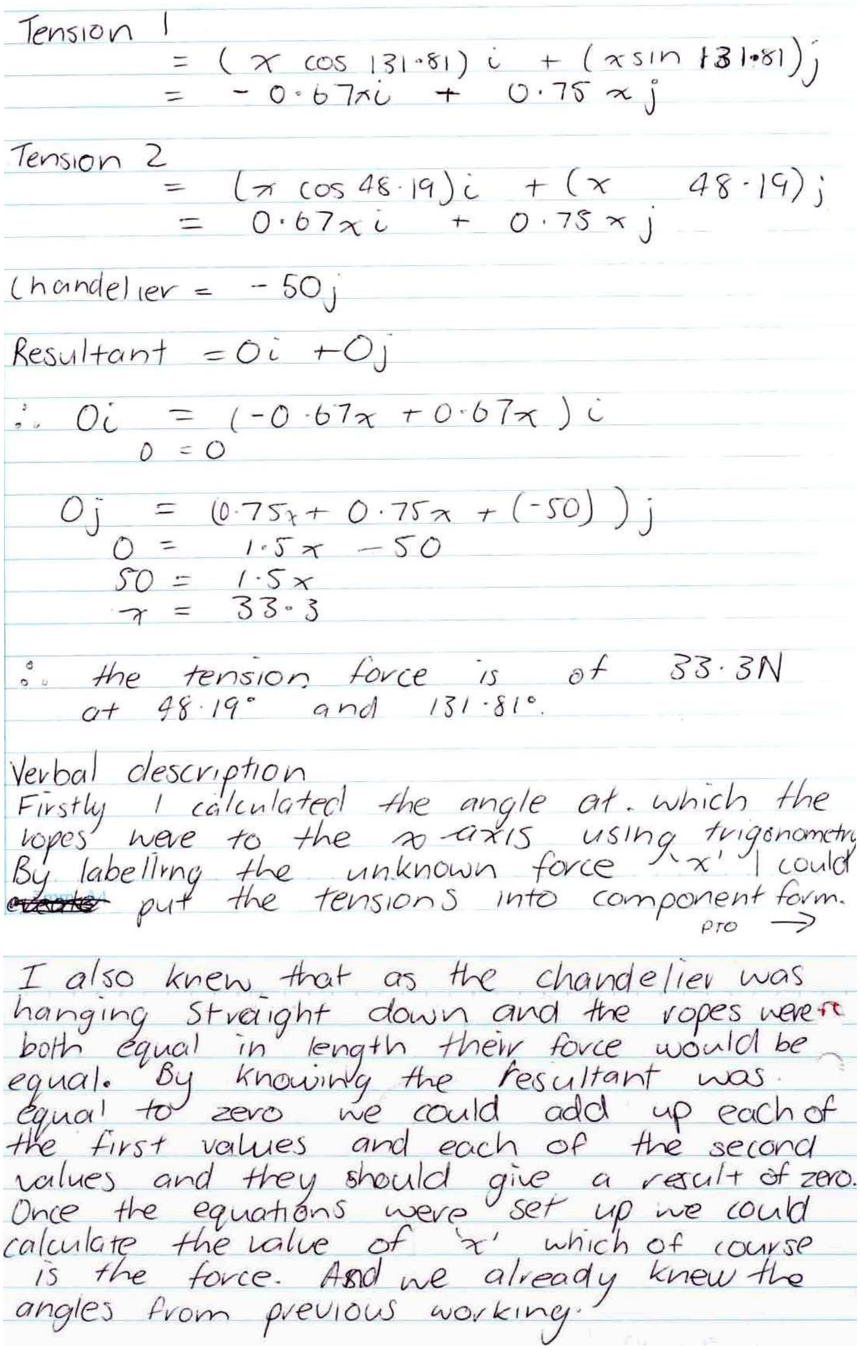


Figure 1: Marco's response to one of my assigned homework questions

As can be seen, Marco's written description goes "I used the vectors in the polar form and put them head to tail to resolve the resultant vector. By writing how I resolved these vectors I discovered they were not in fact *orthogonal*" [italics added]. I thought this was very intriguing for it seems to suggest that the words Marco used to describe his thought processes bore down on his symbolic reasoning and forced him to think critically about what he had done which resulted in him gaining a deeper understanding of the problem, and consequently, identifying his mistake.

During cycle two when I kept up a written conversation with the students, my verbal feedback helped Annie tremendously since she made some errors on each of the three problems given, but was able to answer all three correctly after I returned her solutions with my written explanations of where she had made mistakes. Not only was she able to produce the correct answers, but she also provided accurate verbal descriptions of her reasoning as shown in Figure 2 below. None of the other students were able to give correct solutions in either symbolic or written form even after I had offered them written feedback. The homework questions, along with some student responses, can be found by referring to Appendix A.


Tension 1
= (x cos 131·81) i + (x sin 131·81) j
= − 0·67xi + 0·75 x j

Tension 2
= (x cos 48·19) i + (x 48·19) j
= 0·67x i + 0·75 x j

Chandelier = − 50j

Resultant = 0i + 0j

∴ 0i = (−0·67x + 0·67x) i
0 = 0

0j = (0·75x + 0·75x + (−50)) j
0 = 1·5x − 50
50 = 1·5x
x = 33·3

∴ the tension force is of 33·3N at 48·19° and 131·81°.

Verbal description
Firstly I calculated the angle at which the ropes were to the x-axis using trigonometry. By labelling the unknown force 'x' I could put the tensions into component form.
PTO →

I also knew that as the chandelier was hanging straight down and the ropes were both equal in length their force would be equal. By knowing the resultant was equal to zero we could add up each of the first values and each of the second values and they should give a result of zero. Once the equations were set up we could calculate the value of 'x' which of course is the force. And we already knew the angles from previous working.

Figure 2: Annie's response to one of my assigned homework questions after I had given her written feedback to show her where she made mistakes

Question 12 on the mid-semester 2 exam which was done after the intervention, contained one question which was phrased entirely in words. It stated: "Using the general form for a complex number (x + iy), investigate the validity of the following statement: For any complex number, the conjugate of the multiplicative inverse of the complex number is equal to the multiplicative inverse of the conjugate of the complex number". Here the students needed to rely entirely on their comprehension of the mathematical terms. Unfortunately, none of the students were able to get full marks for this question. Annie did use the correct procedure which gave her a high mark on this problem, but she started off by stating that the *multiplicative identity* is (1 + i), instead of just 1, which kept her from producing the correct answer. Of the others, Marco, Desiree, and Steve were at least able to identify the multiplicative identity as one.

Interestingly, during a review period the students worked on a similar question which asked for the *multiplicative inverse* of (12 – 5i). I had forgotten something in the staff room so I asked the students to work on the problem while I went and retrieved it. However, I left my audio recorder going and the conversation the students had about the problem while I was out was quite insightful. The first question posed by Desiree was "OK, how do you figure that out?", to which Marco added: "What's the multiplicative inverse?". Meredith replied: "Something that's timesed by that [the number (12 – 5i)] which equals one". It is sometimes difficult to follow the conversation due to noise on the audio tape and several of the students talking at the same time, but it is obvious that for roughly the first minute the conversation revolves around the meaning of the relevant terms – in this case, the *multiplicative inverse*. Once that had been established, the students spent the next couple of minutes sharing their ideas about the procedure to find the multiplicative inverse which resulted in Annie arriving at the correct answer, but the others getting distracted and leaving the problem unfinished until I returned.

Question 8 on the mid-semester exam simply asked for the definitions of the terms *consistent* and *dependent* as they relate to a system of equations. Marco and Steve got both wrong, but Chad, Annie, Desiree, and Meredith got one out of the two correct. While this was a somewhat better result than that of question 12 mentioned above, this was a much simpler factual-based question with no higher order thinking skills. Both of these terms were in the students' dictionary and we had spent a couple minutes discussing their significance during the review periods. Perhaps the students did not have enough opportunity to use those terms and integrate them into their active vocabulary.

The assignment which was done during the intervention included two questions in the Modelling and Problem Solving section asking for explanations to an iterative problem. The questions were devoid of any specific equations and thus the students were not to use any calculations, but to give their explanations in words. The questions are numbers 5 and 6 on the assignment and can be found, along with some student responses, by referring to Appendix B. At the beginning of the last class before the assignment was due, the students asked if we could spend a little bit of time discussing these questions since they were all having trouble with them. We started off by reading question 5 together. I needed to reread the question several times before I was sure what the question was asking for. Once I thought I knew the approach which would yield the correct solution, I asked the students what they thought they needed to do. Initially they had very little to say since, as Annie commented in retrospect after the solution started to become clear: “it was because of the wording nobody knew what to do”. I drew a diagram on the board and offered my suggestions as to how we might distinguish between convergent and divergent cases. Once I had introduced the terms *symmetry* and *quadrant* as part of the explanation the students started producing all kinds of ideas as to which cases would produce convergent solutions and which would not. Some of the students produced quite sophisticated explanations to these questions on their assignments using this vocabulary. For example, in response to question 6, Desiree wrote: “if a horizontal line and a vertical line are drawn through the intersection of the two lines, a portion of a line must be in each corner quadrant” as shown in Figure 3 below.

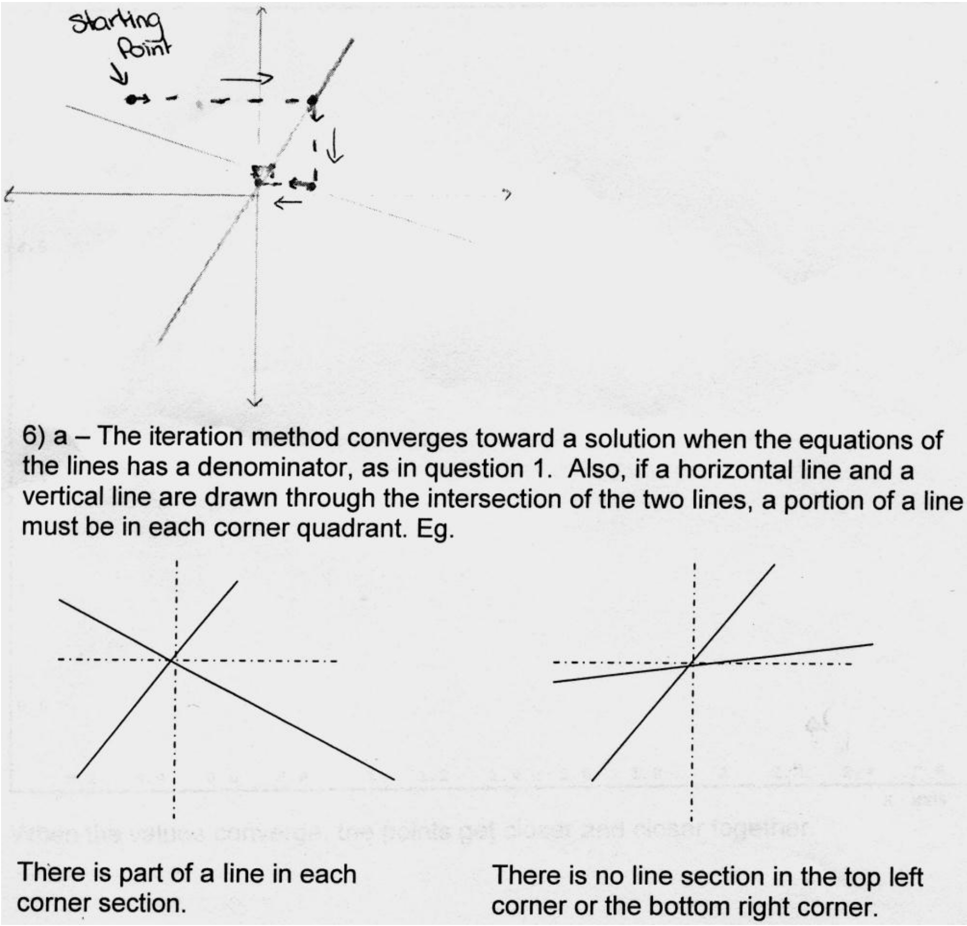


Figure 3: Desiree's response to Question 6 on the assignment

Other comments included that from Marco who wrote: "The iteration method only converges…when the lines of the equation are in the four quadrants of the horizontal and vertical axes of symmetry", and Steve noted: "The iteration method will converge towards a central solution if…there is an angle from the line in every quadrant from the axis of vertical and horizontal symmetry from the intersection point". Reflecting upon this lesson, I recorded in my research journal: "what had started out as a rather reticent group, leapt to a massive exchange of ideas via language and diagrams (no calculations in this problem)" (van Tol, 2008, p. 15).

At the end of the last class I gave the students a feedback form asking them about aspects they liked or disliked about the approach we took to emphasizing the language of mathematics. Their comments strongly supported the method. Some of the statements about what they liked included that from Chad: "often it was easier to understand and less abstract", Desiree: "we contributed to the dictionary so you could remember it better", Meredith: "writing out the dictionary helped me remember the meanings”, Steve: “the questions had become easier to understand than before I knew what some of the mathematical terms meant”, Annie: “it was also good that when you couldn’t solve a problem the verbal feedback helped to find where the error occurred”, and Marco: “we were able to understand some of the language used in mathematical problems and allowed the question to read easier and understand easier what the question meaned [sic]”. Most of the remarks on what they did not like related to the time constraints. For example, Meredith: “we had assignments at the same time, so finding the time to do all the questions was hard”, Annie: “when writing the answer in words it takes time”, and Chad: “maybe under the given time constrains [sic] it took up extra time – but that *isn’t* to say the time went to waste” [emphasis in original].

However, one comment from Desiree was: “some I still didn’t understand”. In response to this, I immediately caught up with her and asked her which terms she was referring to. The term *basis* was still not clear to her so I explained its significance and gave her an example. She was away the day we added that word to the dictionary so perhaps that was the reason for her confusion. Finally, when asked if they would like to continue studying math with an emphasis on terminology, the response was a unanimous ‘yes’. Some of these comments included those from Steve: “I would like to continue using this method because it gave me a great satisfaction in being able to understand terms that I didn’t know before”, Marco: “Yes as it helps to be able to read a question and understand what it is asking”, and Chad: “Yes, it asists [sic] with understanding how different aspects come together. In general it expands our view of everything we learn”.

Discussion

My thoughts when this research began was that the linguistic framework which allows us to speak about math is antecedent to and therefore more fundamental than the mathematical symbolism. It is certainly true from a historical perspective since the Egyptians and even the Greeks, despite their great mathematical achievements, did not have access to the machinery of algebra which was developed by the Arabs much later (Hodgkin, 2005, p. 110). In this way, I hoped that by emphasizing mathematical terminology and encouraging the students to use it appropriately I could help increase their depth of understanding and range of ability.

Since I had hoped that an increase in the students' ability to use mathematical language to express their reasoning would also increase their ability to do math, I thought it would be prudent to try and disprove the opposite; namely, that an increased ability to use mathematical language in expressing one's reasoning does not affect his or her ability to do math. If I could find a student who is able, with fully informed use of appropriate mathematical terminology, to explain his or her thought processes related to solving the problem at hand, but who is unable to solve the problem symbolically – and by that I mean comes up with the wrong answer not because of some arithmetic error, but due to a conceptual misunderstanding, then I would have shown that it is possible that there is no connection between a student's ability to communicate mathematics intelligently, and their ability to carry out a mathematical procedure symbolically.

However, I could not find such evidence since Annie was the only student during cycle 2 who could produce succinct written descriptions of her solutions to problems I gave, which were all accompanied by accurate symbolic reasoning. No student submitted written descriptions detailing their reasoning while being unable to carry out the procedure symbolically. While in a strictly scientific context I would be committing the fallacy of the argument from ignorance (which in its most seductive form states that absence of evidence is not the same as evidence of absence) to suggest that I had discovered some immutable social law, within the context of this action research project it would be careless, perhaps even foolish, not to use this as one piece of evidence in the triangulation of the suggestion that there *is* a relationship between students' ability to use mathematical language and to carry out mathematical computations.

Perhaps arguing that the ability to speak or write intelligently about a topic preceding all other modes of doing seems like a truism since, "the most significant moment in the course of intellectual development, which gives birth to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity…converge" (Vygotsky, 1978, p. 24). Were this indeed such a truism, we might expect there to be a much greater emphasis placed on communicating the reasoning associated with mathematics verbally or in writing. While this is beginning to change, as demonstrated by the inclusion of the assessment category called 'Communication and Justification' in the Queensland mathematics curriculum, its importance does not appear to have filtered down to the general conscience of the public. In my experience, and others would also support this (Archer, 2000, p. 9; Hellman, 2006, p. 214), most people, students or otherwise, still regard mathematics in an abstract, absolutist way, as something that you do purely as symbolism on paper and do not view mathematics as a socio-linguistic phenomenon which developed by people reasoning with one another verbally.

While Fletcher and Santoli found "substantial increases in comprehension of the reasoning behind the math concepts and problems" (2003, p. 2), they had ample time to complete the vocabulary exercises in class. I was under enormous time constraints and as a result I had to move the entire language approach outside of class time during cycle 2. Fortunately, due to the flexibility of the action research methodology this in itself was not problematic. However, the effect of immediate real-time feedback, had I been able to provide it to the students in class, would have at least been more encouraging than when the students were doing their homework by themselves and, as many of them mentioned on the final feedback form, pressed for time by other assignments.

The students all gave support to this approach as per their comments on the final feedback form. Consequently, I would try this method again in another class.

However, as just mentioned, the time restrictions placed significant limitations on this study. I believe that the results could be improved if there was more time in class for the students to practice using the vocabulary to express their understanding, both in writing and by speaking. In addition, in future I think this approach could be improved if I had more critical friends involved in the process. My year 11 class had been amalgamated with a year 12 class due its small size, so when I began teaching my class regularly, Jessica and I agreed to split the classes so each would have its own teacher. While having more individual attention benefited the students, the fact that Jessica could not give me any feedback on my lessons did nothing to benefit my teaching.

The disparity between the students' apparent understanding of the terminology in class and their relatively poor lack of comprehension on the mid-semester 2 exam, as well as the results of the initial vocabulary quiz, suggest that, like most students, their passive vocabulary is notably stronger than their active vocabulary. This is significant for any teacher's praxis since, being lulled into the impression that the students understand the lesson while it is being presented can cause massive short-sightedness. Ensuring that the students can produce both the language and symbolism by themselves must be a continual process.

Finally, though I am supportive of social justice according to each of the definitions given above, recently, I have become aware that in myriad ways the Jesuit definition which accentuates economic relationships is by far the most important. This is not the place to detail why, so a quick example will have to suffice. One day while in the staff room, another teacher lamented the fact that Australia seemed irrevocably headed toward a national curriculum. Initially I thought such hopelessness was mistaken since, politically, Australia is a federation and education is a prerogative of the states. When I pointed this out she replied that while that was true, if the states did not comply the federal government would simply withhold their funding. This is a lucid example of how economic principles (in this case central banking and fiat currency), trump democratic political ones. Thus, if I were to live by my values as action research encourages, I should have worked to effect such economic reform.

Despite my numerous suggestions to both my supervisors to watch an educational video called 'Money as Debt' (Grignon, 2006) which describes the banking system in simple terms, I was only able to secure a period to do so with Dave during the last class for my year 9 science students. The moment of truth came about half way through the video when Dave instructed me to stop the video. When I asked him if he thought that it was unnecessarily subversive or just too brutally honest, he simply replied that he felt it was too much "doom and gloom". The producer Paul Grignon hopes that the video: "will convince… social justice and electoral reform advocates that monetary reform is essential to the goals they hope to accomplish" (n.d., ¶ 12). The reader is encouraged to watch it and decide for him or herself.

The Libertarian teacher John Taylor Gatto has carefully detailed how modern schooling is essentially a consequence of the centralization of the political economy (2006), and advocates parents' and students' right to choose the sort of education they desire. This is in line with Article 26 – 3 of the United Nation's Universal Declaration of Human Rights (2005, ¶ 37). The Nobel prize winning economist and champion of Libertarianism Milton Friedman wrote an article which outlines how a real free market might work to provide such a variety of education with a true *laissez-faire* school system (1998). Even the renowned economist John Kenneth Galbraith, who was often against Libertarian policies wrote:

> In 1969 as these pages were in proof, a new administration took office. It proposed to control more effectively, the supply of money and by thus "fine-tuning" the economy through monetary policy…. The only hope is that, by then, public education will have accomplished in this sphere its oldest and most important function which is that of inculcating a deep and even raucous skepticism of those who, however plausibly, claim supernatural or even exceptional powers not vouchsafed to lesser men (1969, p. 187).

Unfortunately, not only has public education not accomplished this goal, but there does not even seem to be an awareness that monetary reform is, at the risk of sounding overbearing, the single most important facility to freedom in our time[4].

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[4] As I write this, global financial markets are collapsing while the U.S. Congress approved an unprecedented $700 billion U.S. bail-out which will be funded by the citizens of the United States.

Conclusion

Through the use of the action research methodology I attempted to improve my students' mathematical aptitude by focusing on their ability to communicate their reasoning with the pertinent mathematical terminology. The flexibility of the method enabled me to adapt my approach to the problems the students and I encountered. Even though the time limits placed huge constraints on the progress made, the quantitative data suggested a slight increase in the students' capability and much of the qualitative feedback showed that the students felt the technique raised their understanding. However, many of the students were unable to convert their understanding into an increased ability to carry out the mathematical symbolism. My feeling is that this might be improved given more time at both a personal and systematic level.

Fortunately, there is now a greater awareness of the need to support students' linguistic competence in mathematics as evidenced by the curricular policies and literature surveyed. Mathematics education may very well improve as this trend continues. When students describe their understanding in words, it forces them to draw on a different, and in many ways, deeper model of the subject matter. As Chad wrote on his feedback form, this method "expands our view of everything we learn".

Glossary of Mathematical Terms

The following is a list of mathematical terms which appear in the body of this thesis. Many other terms were included in the class dictionary.

Term	Definition
Addition Modulo 5	An operation which returns the remainder of two numbers which are added and then divided by 5. E.g. 6 (*addition modulo 5*) 2 = 3
Associative Law	A rule which states: $(x + y) + z = x + (y + z)$
Asymptote	A line which a curve approaches but never touches
Basis	A set of independent vectors which span a space
Binomial	An expression which contains two terms. E.g. $(3 + x)$, or $(xy^2 – 7z)$
Complex Number	A number which has a real and an imaginary part and usually expressed in the general form: $x + y\mathrm{i}$ where x is the real part and y is the imaginary part
Conjugate	A binomial which has the opposite sign of the second term. E.g. the conjugate of $(x + y\mathrm{i})$ is $(x – y\mathrm{i})$
Consistent	A system of equations is said to be consistent if there is at least one solution to the system
Cyclic Group	A set of numbers which can generated by a single element under the reiteration of a given operation
Dependent	A system of equations is said to be dependent if at least one of the equations can be expressed in terms of the others
Determinant	In the two-by-two matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is defined as $ad – bc$
Discriminant	For a quadratic equation of the general form: $ax^2 + bx + c = 0$, the discriminant is defined as $b^2 – 4ac$
Distributive Law	A rule which states: $a(x + y) = ax + ay$
Generators	A single element which can produce a group of numbers under

	the reiteration of a given operation
Multiplicative Identity	The number 1 since, this is the only number which when multiplied by any other number *a*, gives the result of *a*
Multiplicative Inverse	The multiplicative inverse of some number *a* is written as a^{-1}, and is defined such that $a \times a^{-1} = 1$. E.g. the multiplicative inverse of 2 is $\frac{1}{2}$
Orthogonal	An angle equal to 90°
Polar Form	A coordinate system which defines a point in 2-space by the radius (from the origin) and the angle (from the *x*-axis)
Radian	A angle subtended at the centre of a circle whose arc length is equal to the radius
Resultant Vector	The result of adding two or more vectors together
Surdic Conjugate	A surd is a root of any number which can not be simplified to a rational number (e.g. $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{4}$). The conjugate of a surd follows from the definition given above. E.g. the conjugate of the surd, or the surdic conjugate of 1 + $\sqrt{2}$, is 1 – $\sqrt{2}$
Vector	Any quantity that has both magnitude and direction

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